Factoring Polynomials

In the same way that dividing real numbers "undoes" the process of multiplication, factoring a polynomial separates it into the product of two or more other polynomials.

Polynomial in Standard Form: $6t^2 + 15t - 21 =$

And in Factored Form: $3(2t^2 + 5t - 7)$

Polynomial in Standard Form: $x^2 - 3x + 28 =$

And in Factored Form: $(x + 4)(x - 7)$

Polynomial in Standard Form: $3y^2 - 14y + 16 =$

And in Factored Form: $(3y - 8)(y - 2)$

The Greatest Common Factor

Every monomial term can be written as a product of a real number and one or more variables raised to powers. The **Greatest Common Factor (GCF)** of any polynomial with more than one term is the largest expression that is a factor of each term in the polynomial.

Polynomial in Standard Form: $6t^2 + 15t - 21$

GCF: 3

Factored Form: $3(2t^2 + 5t - 7)$

Polynomial in Standard Form: $15w^4 - 35w^3 - 20w^2$

GCF: $5w^2$

Factored Form: $5w^2(3w^2 - 7w - 4)$

Polynomial in Standard Form: $2xy^2 + 18xy + 2x^2y$

GCF: $2xy$

Factored Form: $2xy(y + 9 + x)$

Polynomial in Standard Form: $3x + 4y$

GCF: 1

Factored Form: $1(3x + 4y)$

*Using the distributive property to multiply the factored form should yield the standard form.

The last example in Polynomial in Standard form: 3x + 4y is a "prime polynomial"—that is, the only factor each of the terms has in common is the number 1. In the standard form, a prime polynomial is said to be "unfactorable".

Factor the GCF from $12t^3 + 27t^2 - 6t$. ____________________________

For more practice, see finding the GCF in OpenStax Elementary Algebra.

Factoring Trinomials of the Form $(\boldsymbol{x^2 + bx + c})$

Our goal is to find integers p and q, so that $(x + p)(x + q)$ is equal to the original polynomial, $x^2 + bx + c$.

The FOIL method of multiplication tells us that $(x + p)(x + q) = x^2 + qx + px + pq$, which means that b must be equal to $q + p$, and c must be equal to pq.

Here is an example: find the factored form of $\boldsymbol{x^2 + 11x + 24}$.

We need p and q so that $11 = q + p$ and $24 = pq$. The most efficient way to start searching for p and q is to list all the ways to factor 24 as a product of integers. Then fill in the list with the sum of each pair of possibilities for p and q.

24 =pq & q +p

- $Multiply\ 1 \cdot 24\ Addition\ 1 + 24 = 25$
- $Multiply\ 2 \cdot 12\ Addition\ \ 2 + 12 = 14$
- $Multiply\ 3 \cdot 8\ \ Addition\ \ 3 + 8 = 11$
- $Multiply\ 4 \cdot 6\ \ Addition\ \ 4 + 6 = 10$
- $Multiply\ -1 \cdot -24\ Addition\ -1 + -24 = -25$
- $Multiply\ -2 \cdot -12\ Addition\ -2 + -12 = -14$
- $Multiply\ -3 \cdot -8\ Addition\ -3 + -8 = -11$
- $Multiply\ -4 \cdot -6\ Addition\ -4 + -6 = -10$

Since $3 \cdot 8 = 24$ and $3 + 8 = 11$, we have found the values of p and q that will give us the factorization:
$x^2 + 11x + 24 = (x + 3)(x + 8)$.

As you search for p and q, keep in mind some helpful hints:

- If $c = pq$ is negative, then *one* of p or q must be negative.
- If $c = pq$ is positive and $b = q + p$ is negative, then both p and q must be negative.

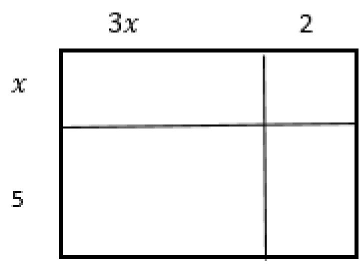
Further examples and practice problems are available at Factoring $x^2 + bx + c$

Find the factored form of $x^2 - 17x + 70$.

Factoring Trinomials of the Form $(ax^2 + bx + c)$

When the leading coefficient of a trinomial is not 1, we must use trial and error or some other method to factor. There are several methods that make it easier to use trial and error, including the area method shown in the example here.

Factor $3x^2 + 17x + 10$ by looking for binomials that represent the lengths of each rectangle:



The areas of each of the smaller rectangles are: $3x^2, 2x, 15x$, and 10. When we add these areas, we get the original polynomial. For more practice, see Factoring trinomials.

Then find the factored form of $4x^2 - 4x - 15$.

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