Linear Equations in Two Variables

Often, we need to relate two quantities, which can be written as ordered pairs. For example,

- Hours worked and dollars earned, (8.5, 110.50)
- Age of a child in months and child's height in inches, (24, 29)
- Years since money was invested and value of the investment, (4.25, 870.79)
- Graduation rate in a certain place and poverty rate, (83%, 31%)
- Daily temperature and kilowatt-hours of electricity used, (84, 21)

The list could go on and on. To display paired information like these examples, we use a Rectangular Coordinate System.

An equation in two variables shows a relationship between two quantities. Here are some examples.

$$y = 3x + 8$$

$$7x - 4y = 42$$

$$x + 15 = 2y + 4$$

The solutions to this type of equation are ordered pairs that make the equation a true statement. In the examples here, all of the order pair solutions to each equation form a line, so we call them *linear equations*. Practice graphing linear equations.

The Slope of a Line

The slope, or how rapidly it is rising or falling, is a key characteristic of a line. If we know any two points on a line, we find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$; that is, slope is the change in y, divided by the change in x. Sometimes this is referred to as "rise over run".

Slope-Intercept Form

When x and y are variables, and m and b are fixed (unchanging) real numbers, then the equation $y = mx + b$ is a linear equation in slope-intercept form. The value m is the slope, and the number b is the y-intercept—the point at which the line passes through the y-axis. Practice using the slope-intercept form.

Finding Equations for Lines

Because any line (that is not vertical) can be described by an equation of the form $y = mx + b$, we can use any of the following information to determine the equation of a line:

- The slope, m, and y-intercept, b
- The slope, m, and any other point on the line
- Any two points on the line

Parallel lines (which never intersect) have equal slopes. Perpendicular lines (which intersect at right angles) have slopes whose product is -1. With this information, we can then write equations for lines that are parallel or perpendicular to each other, if we also know a point on the line.

Learn more, and practice finding equations for lines.

Linear Inequalities

A mathematical statement that uses a less than or greater than symbol tells the reader that two expressions are not equal (in the case of $<$ or $>$) or may not be equal ($\leq$ or $\geq$). Work with linear inequalities and their graphs here.

Practice Problems

1. What is the slope of the line through the points (1,5) and (4,7)?

2. Write an equation for the line through the point (0,8) with slope $m = -2$.

3. Is the ordered pair (-1,-3) a solution to the equation $2x + y = -1$?